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If in (10) we put $1 - 2\psi_1 = 0$, $\psi_1 - 3\psi_1\psi_2 = 0$, $\psi_1\psi_2 - 4\psi_1\psi_2\psi_3 = 0, \ldots\ldots$ then, $\psi_1 = \frac{1}{2}$, $\psi_2 = \frac{1}{3}$, $\psi_3 = \frac{1}{4}, \ldots \psi_{2n-1} = 1 \div 2n$; and these values being constants,

$$\frac{d\psi_1}{dx},\ \frac{d\psi_1}{dh},\ \frac{d\psi_2}{dx},\ \frac{d\psi_2}{dh},\ldots$$

each becomes zero, and equation (10) is *completely satisfied.* Hence, $\psi_1 = \frac{1}{2}$, $\psi_2 = \frac{1}{3}$, $\psi_3 = \frac{1}{4}, \ldots\ldots \psi_{2n-1} = 1 \div 2n$, are always included among the possible values of these fractions in (9), and give

$$\varphi(x+h) = \varphi x + h\varphi' x + \frac{h^2}{2}\varphi'' x + \ldots \frac{h^n}{\underline{|n}}\varphi^n x + \frac{h^{n+1}}{\underline{|n+1}}\varphi^{n+1}x + \ldots \frac{h^{2n}}{\underline{|2n}}\varphi^{2n}x + \ldots \quad \ldots (11)$$

We can also have from (*a*), (*b*), (*c*), . . .

$$\varphi(x+h) = \varphi x + h\varphi' x + \psi_1 h^2\varphi'' x + \psi_1\psi_2 h^3 \varphi''' x + \ldots \psi_1\psi_2 \ldots \psi_{n-1} h^n \times \varphi^n(x+\psi_n h),$$

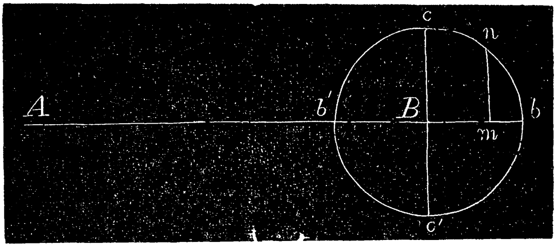
in which, giving $\psi_1, \psi_2 \ldots$ their foregoing values and denoting by ψ_n'' what ψ_n may in consequence become, we get equation (5) in which ψ_n'' has been shown to be a positive proper fraction.

NOTE ON THE HISTORY OF THE METHOD OF LEAST SQUARES.

BY MANSFIELD MERRIMAN, PH. D., NEW HAVEN, CONN.

I.

In the March number of this Journal I stated the first proof of the Method of Least Squares to have been given by Dr. Robert Adrain in 1808 and the second by Gauss in 1809. At that time I had not seen Adrain's original paper and my information concerning it had been entirely derived from an account and partial reprint given by Prof. Cleveland Abbe in the *Amer. Jour. Sci.*, 1871, Vol. I, pp. 411–415. Having lately been able to consult a copy of *The Analyst or Mathematical Museum* in which Adrain's paper was published I found that on pages 96 and 97, to which Prof. Abbe makes no reference, there is given a second deduction of the law of facility of error of an entirely different nature from that presented on pages 93–95. As this is a matter of considerable historical interest and as *The Analyst* for 1808 is quite rare I give the proof in Adrain's own words,



"Suppose that the length and bearing of *AB* are to be measured; and that the little equal straight lines *Bb*, *BC* are the equal probable errors, the one $Bb = Bb'$ of the length of *AB*, and the other $Bc = Bc'$ (perpendicular to the former) of the angle at *A*, when measured on a circular arc to the radius *AB:* and let the question be to find such a curve passing through the four points *b*, *c*, *b'*, *c'*, which are equally distant from *B*, that, supposing the measurement to commence at *A*, the probability of terminating on any point of the curve may be the same as the probability of terminating on any one of the four points *b*, *c*, *b'*, *c'*."

The reasoning following this is long and trivial and ends by concluding that "the curve must be the simplest possible" and that it "must consequently be the circumference of a circle having its centre in *B*." This established, the exponential law of facility is deduced as follows:

"Now let us investigate the probability of the error $Bm = x$, and of $mn = y$. Let *X* and *Y* be two similar functions of *x* and *y* denoting these probabilities, X', Y' their logarithms, then $XY =$ constant, or $X' + Y' =$ constant, and therefore $\dot{X}' + \dot{Y}' = 0$, or $X''\dot{x} + Y''\dot{y} = 0$, whence $X''\dot{x} = -Y''\dot{y}$. But $x^2 + y^2 = r^2 = Bb^2$ therefore $x\dot{x} = -y\dot{y}$, by which dividing $X\dot{x} = -Y\dot{y}$, we have $X'' \div x = Y'' \div y$; and therefore, by a fundamental principle of similar functions, the similar functions $X'' \div x$ and $Y'' \div y$ must be each a constant quantity: put then $X'' \div x = n$, and we have $X''\dot{x} = nx\dot{x}$, that is $\dot{X}' = nx\dot{x}$, and the fluent is $X' = c + \frac{1}{2}nx^2$; in like manner we find $Y' = c + \frac{1}{2}ny^2$, and therefore the probabilities themselves are $e^{c+\frac{1}{2}nx^2}$ and $e^{c+\frac{1}{2}ny^2}$, in which *n* ought to be negative, for the probability of *x* grows less as *x* grows greater."

This proof is the same as that given in 1850 by Herschel (J. F. W.) and usually called Herschel's proof. It is defective in taking $XY =$ constant, that is, in regarding the rectangular deflections *x* and *y* as independent.

It appears then that to Dr. Adrain the second as well as the first proof of the Method of Least Squares must be credited. Why Prof. Abbe, in his otherwise exhaustive account of Adrain's paper, should have overlooked this demonstration is hard to understand, unless we suppose that he accidentally turned two leaves of *The Analyst* at once and thus jumped from the bottom of page 95 to the top of page 98.

II.

I have lately spent (and perhaps wasted) a good deal of labor in making a catalogue or list of all writings relating to the Method of Least Squares and the Theory of accidental Errors of Observation. Books which devote only a page or two to the subject and practical papers in which the Method is used briefly and incidentally I have not cared to include. My aim has been to record every book or paper which can be regarded as a contribution to the science of the Adjustment of Observations, whether written from a theoretical or practical standpoint. A few statistics from this catalogue may perhaps be properly stated here.

The whole number of writings recorded is 404; of these 94 may be classed as books and 310 as memoirs or papers. The earliest bears the date 1722 and the latest 1876. Previous to 1805, the year of Legendre's announcement of the principle of Least Squares there are 22 titles; since 1805 the numbers published in the several decades are

from	1805	to	1814,	18 titles,
	1815		1824,	29
	1825		1834,	33
	1835		1844,	45
	1845		1854,	63
	1855		1864,	71
	1865		1874,	94

showing a steady increase in the interest and importance of the subject.

These books and memoirs are in eight languages, and classified according to the place of publication they fall under twelve countries. The following table shows this classification:

Countries.		*Languages.*	
Germany	152	German	166
France	77	French	109
Great Britian . .	56	English	89
United States . .	33	Latin	15
Belgium	19	Italian	9
Russia	16	Dutch	7
Italy	14	Danish	5
Austria	10	Swedish . . .	4
Switzerland . .	9		
Holland	7		
Sweden	6		
Denmark	5		
Total	404	. . . .	404

These works were written by 192 different authors, 125 of whom produced only one book or paper each. The largest number written by one author is 14; this author was Laplace. Out of the 404 works I have seen and actually inspected 309.

Like all bibliographical researches of this kind my list cannot be regarded as complete. With better library facilities the number of titles in the Italian, Dutch and Scandinavian languages would be much increased; and one who can consult the Russian and Hungarian literature might undoubtedly find a few titles to add although the articles themselves would probably not be of great value. I hope at some future time to be able to publish the list accompanied with historical and critical notes giving brief accounts of the contents of each book or memoir with perhaps an estimate of its value.

PEDAL CURVES.

BY PROF. W. W. JOHNSON, ANNAPOLIS, MARYLAND.

When the rolling curve is equal to the fixed curve, the curves touching at corresponding points, as in the Figure on page 71, No. 3, Analyst, the locus of any point connected with the rolling curve is the same as the Pedal of the given curve, doubled in linear dimensions. For consider the point in the plane of the fixed curve corresponding to the generating point, as D in the figure; it is plain that D and D'' are symmetrical points with reference to the common tangent at P, so that DD'' is perpendicular to the tangent and bisected by it, and the locus of D'' is similar to that of the foot of the perpendicular upon the tangent, 2 being the ratio, and D the centre of similitude.

It may be remarked that since the tangent to the roulette, or locus of D'', and to the pedal are parallel, the well known and obvious property of roulettes that PD'' is a normal affords a simple proof of the property of the pedal that the inclination of its tangent and radius vector is the same as the corresponding angle at the point of contact P in the original curve.

When the equation of the tangent to the given curve, in terms of its direction ratio m, is known, the equation of the pedal is very easily found; for, (h, k) being the pedal origin, the equation of the perpendicular upon the tangent is

$$y - k = -\frac{1}{m}\left(x - h\right),$$

hence if we eliminate m between this equation and that of the tangent, by substituting in the latter $m = (h - x) \div (y - k)$, we shall at once have the quation of the pedal.